Mathematical analysis 2, WNE, 2018/2019
meeting 12. – homework solutions

28 March 2019

Group 8:00

Check whether

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\frac{1}{x^2+y^2} & \text{, for } (x,y) \neq (0,0) \\ 0 & \text{, for } (x,y) \neq (0,0) \end{cases}$$

Is differentiable at $(0,0)$. Calculate partial derivatives at this point.

Partial derivatives

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h^2}}{h} = \lim_{h\to 0} h \sin\frac{1}{h^2} = 0,$$

because $\sin\frac{1}{h^2}$ is bounded. Similarly,

$$\frac{\partial f}{\partial y}(0,0) = 0.$$

We check the limit

$$\lim_{(h_x,h_y)\to 0} \frac{(h_x^2+h_y^2)\sin\frac{1}{(h_x^2+h_y^2)} - 0 - 0}{\sqrt{h_x^2+h_y^2}} = \lim_{(h_x,h_y)\to 0} \sqrt{h_x^2+h_y^2}\sin\frac{1}{(h_x^2+h_y^2)} = 0.$$

So it is differentiable and $Df_{(0,0)}(h_x,h_y) = 0$.

Group 9:45

Check whether the function

$$f(x,y) = \sqrt[3]{x^3+y^3}$$

Is differentiable at $(0,0)$. Calculate partial derivatives at this point.

Partial derivative

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{\sqrt[3]{h^3}}{h} = 1,$$

Similarly,

$$\frac{\partial f}{\partial y}(0,0) = 1.$$

We check the limit

$$\lim_{(h_x,h_y)\to 0} \frac{\sqrt[3]{h_x^3+h_y^3} - h_x - h_y}{\sqrt{h_x^2+h_y^2}}.$$

For $h_x = h_y = 1/n$ we get

$$\lim_{n\to\infty} \frac{(\sqrt[3]{2}-1)/n}{\sqrt{2}/n} = \frac{(\sqrt[3]{2}-1)}{\sqrt{2}} \neq 0,$$

the limit does not exists so the function is not differentiable at $(0,0)$.