

Mathematical analysis 2, WNE, 2018/2019 meeting 11.

26 March 2019

Problems

1. Find all functions f , such that

$$\begin{aligned}\frac{\partial f}{\partial x}(x, y) &= 2xy^3 + e^x \sin y, \\ \frac{\partial f}{\partial y}(x, y) &= 3x^2y^2 + e^x \cos y + 1.\end{aligned}$$

2. Find the directional derivative of the above function at point $(0, 0)$ in direction $v = (2, 1)$.
3. Examine the differentiability of:

a) $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{for } (x, y) \neq (0, 0) \\ 0, & \text{for } (x, y) = (0, 0) \end{cases},$

b) $f(x_1, \dots, x_k) = \sqrt{x_1^2 + \dots + x_k^2},$

c) $f(x, y) = \begin{cases} \frac{\sin xy}{y}, & \text{for } y \neq 0 \\ x, & \text{for } y = 0 \end{cases}.$

d) $f(x, y, z) = \sqrt{xyz}$

4. Calculate partial derivatives of $g \circ f$:

a) $f: \mathbb{R}^+ \rightarrow \mathbb{R}^2, f(x) = (x, \sqrt{x}), g: \mathbb{R}^2 \rightarrow \mathbb{R}, g(a, b) = e^{-(a^2+b^2)},$

b) $f: \mathbb{R} \rightarrow \mathbb{R}^2, f(x) = (\cos x, \sin x), g: \mathbb{R}^2 \setminus \{(0, 0)\} \rightarrow \mathbb{R}, g(a, b) = \frac{1}{a^2 + b^2},$

c) $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2, f(x, y) = (x - y, x + y), g: \mathbb{R}^2 \rightarrow \mathbb{R}^2, g(a, b) = (e^a \cos b, e^a \sin b).$

Homework

Group 8:00

Calculate the derivative of $g \circ f$, for $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3, f(x, y) = (x^2 - y^2, x^2 + y^2, x^2y^2), g: \mathbb{R}^3 \rightarrow \mathbb{R}, g(a, b, c) = ab + bc + ac.$

Group 9:45

Calculate the derivative of $g \circ f$, for $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3, f(x, y) = (x - y, x + y, 2\sqrt{xy}), g: \mathbb{R}^3 \setminus \{0\} \rightarrow \mathbb{R}, g(a, b, c) = \ln(a^2 + b^2 + c^2).$