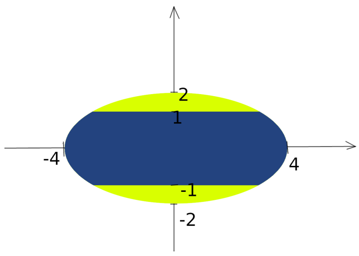
Mathematical analysis 2, WNE, 2018/2019
meeting 10. – short test – solutions

21 March 2019

A

1. Sketch the set
$$\{(x,y) \in \mathbb{R}^2 \colon x^2 + 4y^2 \leqslant 16 \wedge |y| \leqslant 1\}$$
and determine whether it is bounded, convex, open, closed, or compact.

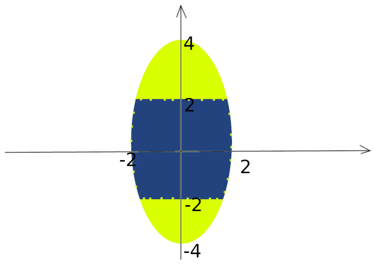


It is bounded, convex, closed and compact, but not open.

2. Check whether there exists
$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^2}{x^2 + y}.$$
If it does, calculate it.
Yes,
$$\frac{x^4 - y^2}{x^2 + y} = \frac{(x^2 - y)(x^2 + y)}{x^2 + y} = x^2 - y \to 0$$

B

1. Sketch the set
$$\{(x,y) \in \mathbb{R}^2 \colon 4x^2 + y^2 < 16 \wedge |y| < 2\}$$
and determine whether it is bounded, convex, open, closed, or compact.



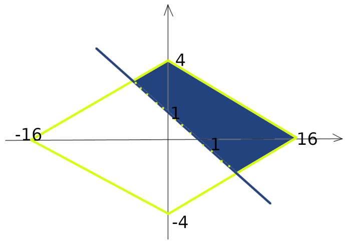
It is bounded, convex and open, but neither closed nor compact.

2. Check whether there exists
$$\lim_{(x,y)\to(0,0)} \frac{y^4 - x^4}{x^2 + y^2}.$$
If it does, calculate it.

Yes,
$$\frac{y^4 - x^4}{x^2 + y^2} = \frac{(y^2 - x^2)(x^2 + y^2)}{x^2 + y^2} = y^2 - x^2 \to 0$$

C

1. Sketch the set
$$\{(x, y) \in \mathbb{R}^2 : |x| + |4y| < 16 \wedge x + y > 1\}$$
and determine whether it is bounded, convex, open, closed, or compact.



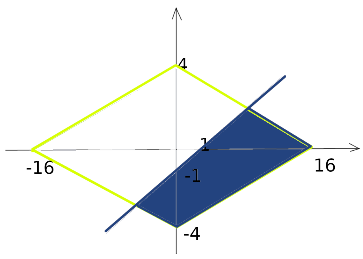
It is bounded, convex and open, but neither closed nor compact.

2. Check whether there exists
$$\lim_{(x,y)\to(0,0)} \frac{2x^2 - y^2}{x^3 + y^3}.$$
If it does, calculate it.

The limit does not exist, because for $x_n = 1/n$, $y_n = 0$.
$$\frac{2x^2 - y^2}{x^3 + y^3} = \frac{2/n^2}{1/n^3} = 2n \to \infty.$$
But for $x_n = 0$, $y_n = 1/n$.
$$\frac{2x^2 - y^2}{x^3 + y^3} = \frac{-1/n^2}{1/n^3} = -n \to -\infty.$$

D

1. Sketch the set
$$\{(x, y) \in \mathbb{R}^2 : |x| + |4y| \leqslant 16 \wedge x - y \geqslant 1\}$$
and determine whether it is bounded, convex, open, closed, or compact.



It is bounded, convex, closed and compact, but not open.

2. Check whether there exists

$$\lim_{(x,y)\to(0,0)} \frac{3y^2 - x^2}{x^3 + y^3}.$$

If it does, calculate it.

The limit does not exists, because for $x_n = 1/n$, $y_n = 0$.

$$\frac{3y^2 - x^2}{x^3 + y^3} = \frac{-1/n^2}{1/n^3} = -n \to -\infty.$$

But for $x_n = 0$, $y_n = 1/n$.

$$\frac{3y^2 - x^2}{x^3 + y^3} = \frac{3/n^2}{1/n^3} = 3n \to \infty.$$