Mathematical analysis 2, WNE, 2018/2019
meeting 10. – solutions

21 March 2019

1. Find the limits
$$\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$
and
$$\lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$
for

a) $f(x,y)=x+y$,
$$\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}=\lim_{h\to 0}\frac{x+h+y-x-y}{h}=1.$$
$$\lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}=\lim_{h\to 0}\frac{x+y+h-x-y}{h}=1.$$

b) $f(x,y)=x^2y^3-2x$.
$$\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}=\lim_{h\to 0}\frac{y^3(x^2+2xh+h^2)-2(x+h)-y^3x^2+2x}{h}=$$
$$=\frac{y^3(2xh+h^2)-2h}{h}=2xy^3-2.$$
$$\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}=\lim_{h\to 0}\frac{x^2(y^3+3y^2h+3yh^2+h^3)-2x-y^3x^2+2x}{h}=$$
$$=\frac{x^2(3y^2h+3yh^2+h^3)}{h}=3x^2y^2.$$

2. Let
$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & \text{, for } (x,y)\neq(0,0)\\ 0 & \text{, for } (x,y)=(0,0)\end{cases}.$$
Check whether there exist $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.
$$\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}=\lim_{h\to 0}\frac{0h/h^2-0}{h}=0.$$
Thus, there exists $\frac{\partial f}{\partial x}(0,0)$, and similarly $\frac{\partial f}{\partial y}(0,0)$ because x and y can be interchanged here.

3. Find the partial derivatives of functions:

a) $f(x,y)=x^y$,
$$\frac{\partial f}{\partial x}=yx^{y-1}.$$
$$\frac{\partial f}{\partial y}=x^y\ln x.$$

b) $f(x,y,z) = x^2y^3z^4$,

$$\frac{\partial f}{\partial x} = 2xy^3z^4.$$
$$\frac{\partial f}{\partial y} = 3x^2y^2z^4.$$
$$\frac{\partial f}{\partial z} = 4z^2y^3z^3.$$

c) $f(x,y,z) = e^{xyz}$,

$$\frac{\partial f}{\partial x} = yze^{xyz}.$$
$$\frac{\partial f}{\partial y} = xze^{xyz}.$$
$$\frac{\partial f}{\partial z} = xye^{xyz}.$$

d) $f(x,y,z) = xe^y + ye^z + ze^x$.

$$\frac{\partial f}{\partial x} = e^y + ze^x.$$
$$\frac{\partial f}{\partial y} = xe^y + e^z.$$
$$\frac{\partial f}{\partial z} = ye^z + e^x.$$