# Mathematical analysis 2, WNE, 2018/2019
# meeting 3. – homework solutions

26 February 2019

## Group 8:00

Calculate:

$$\int 2\frac{e^{10x}+4e^{8x}+9e^{6x}+13e^{4x}+10e^{2x}}{(e^{2x}+3)\left(e^{4x}+2e^{2x}+2\right)^2}\,dx.$$

Hint: $\frac{t^4+4t^3+9t^2+13t+10}{(t+3)(t^2+2t+2)^2}=\frac{A}{t+3}+\frac{Bx+C}{(t^2+2t+2)^2}$.

$$\int 2\frac{e^{10x}+4e^{8x}+9e^{6x}+13e^{4x}+10e^{2x}}{(e^{2x}+3)\left(e^{4x}+2e^{2x}+2\right)^2}\,dx \overset{\boxed{t=e^{2x},\ \frac{dt}{dx}=2e^{2x}}}{=}$$

$$=\int \frac{t^4+4t^3+9t^2+13t+10}{(t+3)(t^2+2t+2)^2}\,dt \overset{\boxed{A=1,B=1,C=2}}{=}$$

$$\int \frac{t+2}{(t^2+2t+2)^2}\,dt+\int\frac{dt}{t+3}=$$

$$=\frac{1}{2}\int\frac{2t+2}{(t^2+2t+2)^2}\,dt+\int\frac{dt}{(t^2+2t+2)^2}+\ln|t+3| \overset{\boxed{\Delta=-4,p=2,u=t+1,\ \frac{dt}{du}=1}}{=}$$

$$-\frac{1}{2(t^2+2t+2)}+\int\frac{du}{(1+u^2)^2}+\ln|t+3|=-\frac{1}{2(t^2+2t+2)}+\frac{u}{2(1+u^2)}+\frac{1}{2}\int\frac{du}{1+u^2}+\ln|t+3|=$$

$$-\frac{1}{2(t^2+2t+2)}+\frac{u}{2(1+u^2)}+\frac{1}{2}\text{arctg}u+\ln|t+3|+C=$$

$$=-\frac{1}{2(t^2+2t+2)}+\frac{t+1}{2(1+(t+1)^2)}+\frac{1}{2}\text{arctg}(t+1)+\ln|t+3|+C=$$

$$=-\frac{1}{2(e^{4x}+2e^{2x}+2)}+\frac{e^{2x}+1}{2(1+(e^{2x}+1)^2)}+\frac{1}{2}\text{arctg}(e^{2x}+1)+\ln(e^{2x}+3)+C$$

## Group 9:45

Calculate:

$$\int \frac{e^{5x}+4e^{4x}+9e^{3x}+13e^{2x}+10e^{x}}{(e^{x}+3)\left(e^{2x}+2e^{x}+2\right)^2}\,dx.$$

Hint: $\frac{t^4+4t^3+9t^2+13t+10}{(t+3)(t^2+2t+2)^2}=\frac{A}{t+3}+\frac{Bx+C}{(t^2+2t+2)^2}$.

$$\int \frac{e^{5x}+4e^{4x}+9e^{3x}+13e^{2x}+10e^{x}}{(e^{x}+3)\left(e^{2x}+2e^{x}+2\right)^2}\,dx \overset{\boxed{t=e^{x},\ \frac{dt}{dx}=e^{x}}}{=} \int \frac{t^4+4t^3+9t^2+13t+10}{(t+3)(t^2+2t+2)^2}\,dt \overset{\boxed{A=1,B=1,C=2}}{=}$$

$$\int \frac{t+2}{(t^2+2t+2)^2}\,dt+\int\frac{dt}{t+3}=$$

$$= \frac{1}{2}\int \frac{2t+2}{(t^2+2t+2)^2}\,dt + \int \frac{dt}{(t^2+2t+2)^2} + \ln|t+3| \overset{\boxed{\Delta=-4, p=2, u=t+1, \frac{dt}{du}=1}}{=}$$

$$-\frac{1}{2(t^2+2t+2)} + \int \frac{du}{(1+u^2)^2} + \ln|t+3| = -\frac{1}{2(t^2+2t+2)} + \frac{u}{2(1+u^2)} + \frac{1}{2}\int \frac{du}{1+u^2} + \ln|t+3| =$$

$$-\frac{1}{2(t^2+2t+2)} + \frac{u}{2(1+u^2)} + \frac{1}{2}\mathrm{arctg} u + \ln|t+3| + C =$$

$$= -\frac{1}{2(t^2+2t+2)} + \frac{t+1}{2(1+(t+1)^2)} + \frac{1}{2}\mathrm{arctg}(t+1) + \ln|t+3| + C =$$

$$= -\frac{1}{2(e^{2x}+2e^x+2)} + \frac{e^x+1}{2(1+(e^x+1)^2)} + \frac{1}{2}\mathrm{arctg}(e^x+1) + \ln(e^x+3) + C$$