Linear Algebra, WNE, 2018/2019
meetings 27.

24 January 2019

Problems

1. Consider the following system of equations

$$\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 1 \\ x_2 + 2x_2 + 3x_3 + x_4 = 3 \\ 3x_1 + 5x_2 + 8x_3 + tx_4 = 9 \\ 3x_1 + 4x_2 + tx_3 + 3x_4 = 5 \end{cases}.$$

   (a) For which real numbers $t \in \mathbb{R}$ this system is consistent?

   (b) For which real numbers $t \in \mathbb{R}$ this system has exactly one solution?

2. Let $V = \text{lin}((1, 1, 2, 3), (2, 3, 5, 7), (5, 6, 11, 16))$.

   (a) Find a basis and the dimension of V.

   (b) For which real numbers $t \in \mathbb{R}$, $V = \text{lin}((1, 1, 2, 3), (2, 3, 5, 7), (5, 6, 11, 16), (1, 0, 1, t))$?

3. Let $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 5x_1 + 2x_2 - x_3 = 0\}$ and let $\alpha_1 = (1, 0, 5), \alpha_2 = (1, 2, 9)$.

   (a) Give an example of vector α_3, such that system of vectors $\alpha_1, \alpha_2, \alpha_3$ is a basis of $\mathbb{R}^3$ and $3, 4, 1$ are the coordinates of $\beta = (9, 9, 56)$ in this basis.

   (b) Does there exist a vector $\gamma \in V$ such that the system $\alpha_1, \alpha_2, \gamma$ is a basis of $\mathbb{R}^3$? If so, give an example of such a vector γ. If not, explain why such a vector γ does not exist.

4. Assume that the matrix of $\varphi : \mathbb{R}^3 \to \mathbb{R}^2$ has in bases $\mathcal{A} = \{(0, 1, 2), (0, 0, 1), (1, 1, 3)\}$ and $\mathcal{B} = \{(2, 1), (1, 0)\}$ matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 4 \\ 6 & 1 & 4 \end{bmatrix}$.

   (a) Calculate $\varphi((0, 1, 0))$.

   (b) Find a matrix of φ in bases $\mathcal{C} = \{(1, 1, 1), (1, 2, 3), (2, 1, 1)\}$ and $\mathcal{D} = \{(0, 1), (1, 1)\}$.

5. Let $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$, and let $A^{150} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

   (a) Check whether matrix A is diagonalizable. If so, find diagonal matrix similar to A.

   (b) Calculate x.

6. Consider hyperplane $H \subseteq \mathbb{R}^4$, $H = (1, 2, 1, 1) + \text{lin}((1, 1, 0, 2), (1, 2, 0, 3), (1, 1, 1, 4))$ and a line $L \subseteq \mathbb{R}^4$ going though $(1, 0, 1, 0)$ and $(3, 1, 2, 4)$.

   (a) Find and equation describing H.

   (b) Find a parametrization of L and the point of intersection of L and H.

7. Consider the following linear programming problem $4x_1 + x_2 + 2x_3 + x_4 + 5x_5 \to \min$ with constraints:

$$\begin{cases} 2x_1 + x_2 + x_3 + x_5 = 2 \\ 3x_1 + x_2 + 3x_3 + x_4 + 4x_5 = 7 \\ x_1, x_2, x_3, x_4, x_5 \geqslant 0 \end{cases}.$$

   (a) Find whether $\{2, 4\}$ is a feasible set of basic variables.

   (b) Solve this problem using simplex method.

8. Consider quadratic forms $q_1 : \mathbb{R}^3 \to \mathbb{R}$, $q_1(x_1, x_2, x_3) = x_1^2 + 5x_2^2 + 7x_3^2 + 4x_1x_2 - 2x_1x_3$ and $q_2 : \mathbb{R}^3 \to \mathbb{R}$, $q_2(x_1, x_2, x_3) = 2x_1x_2 + 2x_2x_3$.

   (a) Check whether q_1 is positively definite?

   (b) Check whether q_2 is negatively semidefinite?