Linear Algebra, WNE, 2018/2019
meeting 1. – solutions

2 October 2018

1. Mark on the plane $\mathbb{R}^2$ the sets of solutions to the following equations:

   - $2x + 3y = 6$,
     This is the line L meeting points $(0,2)$ and $(3,0)$.
   - $ax + 3y = 0$ depending on parameter $a \in \mathbb{R}$,
     It is a line meeting the point $(0,0)$ and tilted in such a way that it meets point $(1, \frac{-a}{3})$.
   - $2x + 3y = c$ depending on parameter $c \in \mathbb{R}$.
     It is a line parallel to L and meeting point $(\frac{c}{2}, 0)$.

   How many are there, and what are the solutions (depending on parameters $a, c \in \mathbb{R}$) of the following systems of equations:

   - $\begin{cases} 2x + 3y = 6 \\ ax + 3y = 0 \end{cases}$,
     If $a = 2$ this system does not have any solution, so the set of solutions is empty. Otherwise, it has exactly one solution $(\frac{6}{2-a}, \frac{2a}{a-2})$.
   - $\begin{cases} 2x + 3y = 6 \\ 2x + 3y = c \end{cases}$.
     If $c = 6$ the set of solutions is exactly the line L. Otherwise, the system does not have any solution.

2. Solve (e.g. calculating subsequent variables by substitution) the following systems of equations:

   - $\begin{cases} x + 2y - z = 6 \\ 2x + 5y - 2z = 2 \\ x + 3y + z = 5 \end{cases}$
     $x = 6 - 2y + z$, we get:
     $$\begin{cases} 12 - 4y + 2z + 5y - 2z = 2 \\ 6 - 2y + z + 3y + z = 5 \end{cases},$$
     therefore,
     $$\begin{cases} y = -10 \\ y + 2z = -1 \end{cases},$$
     thus, $2z = -1 + 10$, so $z = \frac{9}{2}$ and $x = \frac{61}{2}$.
   - $\begin{cases} x + y + w + z = 4 \\ 2x + y + w + z = 9 \\ x + 2y + w + z = 9 \\ x + y + 2w + z = 10 \end{cases}$
     Subtracting the first equation from the second one we get that $x = 5$. Subtracting the first from the third, we get $y = 5$, and similarly, subtracting the first from the forth, we get $w = 6$. Thus, $16 + z = 4$, so $z = -12$.

3. Find all the roots of polynomial $-2x^4 - 4x^3 + 32x^2 + 4x - 30$.

   We immediately guess that 1 is a root of this polynomial. Therefore, we divide it by $(x-1)$ (e.g. in the usual method). We get $-2x^3-6x^2+26x+30$ (because $(-2x^3-6x^2+26x+30)\cdot(x-1) = -2x^4-4x^3+32x^2+4x-30$). Next, we guess that also -1 is a root and we divide the polynomial by $x+1$, getting $-2x^2 - 4x + 30$. Now by usual method, we get that the other roots are -5 and 3.