Linear algebra, WNE, 2018/2019
meeting 26.

22 January 2019

Problems

1. Check whether the following quadratic forms are positive definite or negative definite
   - $q\colon \mathbb{R}^2 \to \mathbb{R}, q((x_1, x_2)) = -x_1^2 + 4x_1x_2 - 5x_2^2$,
   - $q\colon \mathbb{R}^2 \to \mathbb{R}, q((x, y, z)) = x^2 + 2y^2 + 2z^2 + 2xy + 2xz$.
   - $q\colon \mathbb{R}^4 \to \mathbb{R}, q((a, b, c, d)) = a^2 + 3b^2 + 5c^2 + 7d^2 + 2ab + 2ac + 2ad + 2bc + 6bd + 4cd$.
2. For which real numbers $r \in \mathbb{R}$ the quadratic form $q\colon \mathbb{R}^3 \to \mathbb{R}, q((x, y, z)) = -x^2 + ry^2 + rz^2 + 4xy + 2yz$ is negatively definite?
3. Using eigenvalues check whether the following quadratic forms are positively or negatively definite, positively or negatively semidefinite, or indefinite.
   - $q\colon \mathbb{R}^2 \to \mathbb{R}, q((x, y)) = x^2 + 9y^2 + 6xy$,
   - $q\colon \mathbb{R}^4 \to \mathbb{R}, q((x, y, z, t)) = 5x^2 + 5y^2 + 4z^2 + t^2 + 6xy + 4zt$.
4. For which real numbers $r, s \in \mathbb{R}$ the quadratic form $q\colon \mathbb{R}^3 \to \mathbb{R}, q((x, y, z)) = x^2 + 2rxy + 4y^2 + sz^2$ is:
   - positively definite?
   - positively semidefinite?
   - negatively definite?
   - negatively semidefinite?
   - indefinite?