Linear algebra, WNE, 2018/2019
meeting 21. – solutions

13 December 2018

1. Find a system of linear equations describing:

   - an affine subspace with direction $\text{lin}((1,3,0,1),(2,9,4,2)) \subseteq \mathbb{R}^4$ going through $(1,1,-1,2)$,
   - hyperplane $(1,4,-3,2)+\text{lin}((1,2,0,-3),(1,4,-2,-3),(0,3,-1,-2))$.

   - First we find a system of equation describing the direction
   $$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 9 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -4 & 1 \\ 0 & 1 & \frac{4}{3} & 0 \end{bmatrix},$$
   so a basis of its solutions is $(12,-4,3,0),(-1,0,0,1)$, and we get a system describing the direction
   $$\begin{cases} 12a-4b+3c=0 \\ -a+d=0 \end{cases}$$
   Using the point we calculate the free coefficients $12-4-3=5$ and $-1+2=1$, so finally we get
   $$\begin{cases} 12a-4b+3c=5 \\ -a+d=1 \end{cases}$$
   - First, we find a system of equations for the direction
   $$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 1 & 4 & -2 & -3 \\ 0 & 3 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$
   we get a basis $(1,1,1,1)$, so the direction is described by $a+b+c+d=0$. The free coefficient is $1+4-3+2=4$, so finally we get $a+b+c+d=4$.

2. Find a system of equations describing

   - plane $M \subseteq \mathbb{R}^3$ going through $(6,1,-3),(1,5,1),(1,8,2)$,
   - line $L \subseteq \mathbb{R}^3$ going through $(1,2,-1),(3,4,2)$.

   - $M=(6,1,-3)+\text{lin}((-5,4,4),(-5,7,5))$. And similarly as before we start by finding a system of equations for the direction
   $$\begin{bmatrix} -5 & 4 & 4 \\ -5 & 7 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & \frac{8}{5} & 0 \\ 0 & 3 & 1 \end{bmatrix},$$
   we get a basis $(-8,5,-15)$, so the direction is given by equation $-8x+5y-15z=0$. The free variable is $-48+5+45=2$, so we get equation $-8x+5y-15z=2$.
   - $M=(1,2,-1)+\text{lin}((2,2,3))$. We get the following basis for the coefficients of the system of equations $(-1,1,0),(-3,0,2)$, so the direction is described by
   $$\begin{cases} -x+y=0 \\ -3x+2z=0 \end{cases}$$
   The free coefficient is $-1+2=1$ and $-3-2=-5$, so finally we get
   $$\begin{cases} -x+y=1 \\ -3x+2z=-5 \end{cases}$$

3. Find a parametrization of

   - line $L \subseteq \mathbb{R}^3$ going through $(1,1,5),(3,2,4)$,
   - plane $P \subseteq \mathbb{R}^3$ described by equation $2x_1 + 5x_2 - x_3 = 7$,
   - hyperplane $H \subseteq \mathbb{R}^4$ described by equation $x + y - 3z + 2t = 5$.

   - $L = (1,1,5) + \mathrm{lin}((2,1,-1))$, so the parametrization is $(1+2a, 1+a, 5-a)$.
   - We get the following general solution $(x_1, x_2, 2x_1 + 5x_2 - 7)$, and this is a parametrization.
   - We get the following general solution $(5 - y + 3z - 2t, y, z, t)$, and this is a parametrization.

4. Find a system of equations and a parametrization of

   - line $L \subseteq \mathbb{R}^3$ going through $(2,1,1)$ and perpendicular to the plane described by equation $3x - y + 2z = 6$,
   - plane $M \subseteq \mathbb{R}^3$ going through $(3,0,5)$ and perpendicular to the line $(1,1,1) + \mathrm{lin}((2,-1,1))$.

   - Therefore $\vec{L}^{\perp}$ is described by $3x - y + 2z = 0$, so it is spanned by $(1,3,0),(0,2,1)$, so $\vec{L}$ is described by
     $$\begin{cases} x + 3y = 0 \\ 2y + z = 0 \end{cases}$$
     We calculate the free coefficients $2+3=5$ and $2+1=3$, so we get the following system of equations
     $$\begin{cases} x + 3y = 5 \\ 2y + z = 3 \end{cases}$$
     Therefore the parametrization is $(5 - 3y, y, 3 - 2y)$.
   - Hence, $\vec{M}$ is described by $2x - y + z = 0$, and the free coefficient equals $6 + 5 = 11$, so M is described by $2x - y + z = 11$, and the parametrization is $(x, 2x + z - 11, z)$.