Linear algebra, WNE, 2018/2019
meeting 21. – homework solutions

13 December 2018

Group 8:00

1. Find a system of equations and a parametrization of a hyperplane in $\mathbb{R}^4$ going through

$$(1,0,1,1),(2,5,3,0),(2,2,1,1),(0,1,2,3).$$

This hyperplane is $(1,0,1,1)+\text{lin}((1,5,2,-1),(1,2,0,0),(-1,1,1,2))$. So we immediately get a parametrization

$$\{(1+t+u-w,5t+2u+w,1+2t+w,1-t+2w)\colon t,u,w\in\mathbb{R}\}.$$

We calculate the coefficients of a system of equations for the direction

$$\begin{bmatrix} 1 & 5 & 2 & -1 \\ 1 & 2 & 0 & 0 \\ -1 & 1 & 1 & 2 \end{bmatrix} \to \ldots \to \begin{bmatrix} 1 & 0 & 0 & -\frac{10}{3} \\ 0 & 1 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

We get basis $\{(\frac{10}{3},-\frac{5}{3},3,1)\}$, so the direction is described by $10x_1-5x_2+9x_3-3x_4=0$, and the free coefficient is $10+9-3=16$, so we get the following equation describing the hyperplane

$$10x_1-5x_2+9x_3-3x_4=16.$$

2. Find a parametrization of the plane in $\mathbb{R}^4$ which goes through $(3,1,2,1)$ and is parallel to

$$H:\begin{cases} x_1+x_2-2x_3+x_4=2 \\ 2x_1+3x_2-2x_3+x_4=3 \end{cases}$$

We get immediately a system of equations for the direction

$$\begin{cases} x_1+x_2-2x_3+x_4=0 \\ 2x_1+3x_2-2x_3+x_4=0 \end{cases}$$

And the free coefficients are $3+1-4+1=1, 6+3-4+1=6$, so we get a system for the plane

$$\begin{cases} x_1+x_2-2x_3+x_4=1 \\ 2x_1+3x_2-2x_3+x_4=6 \end{cases}.$$

To find a parametrization we have to solve it

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 1 \\ 2 & 3 & -2 & 1 & 6 \end{bmatrix} \to \ldots \to \begin{bmatrix} 1 & 0 & -4 & 2 & -3 \\ 0 & 1 & 2 & -1 & 4 \end{bmatrix}$$

So the general solution (and a parametrization) is:

$$\{(-3+4x_3-2x_4,4-2x_3+x_4,x_3,x_4)\colon x_3,x_4\in\mathbb{R}\}$$

Group 9:45

1. Find a system of equation and a parametrization of a hyperplane in $\mathbb{R}^4$ going through

$$(-1,0,-1,-1),(-2,-5,-3,0),(-2,-2,-1,-1),(0,-1,-2,-3).$$

This hyperplane is $(-1,0,-1,-1)+\text{lin}((-1,-5,-2,1),(-1,-2,0,0),(1,-1,-1,-2))$. So we immediately get a parametrization

$$\{(-1-t-u+w,-5t-2u-w,-1-2t-w,-1+t-2w)\colon t,u,w\in\mathbb{R}\}.$$

So we calculate the coefficients for the system of equations describing the direction

$$\begin{bmatrix} -1 & -5 & -2 & 1 \\ -1 & -2 & 0 & 0 \\ 1 & -1 & -1 & -2 \end{bmatrix} \to \ldots \to \begin{bmatrix} 1 & 0 & 0 & -\frac{10}{3} \\ 0 & 1 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

getting a basis $\{(\frac{10}{3},-\frac{5}{3},3,1)\}$, so the directions described by $10x_1-5x_2+9x_3-3x_4=0$, and the free coefficient is $-10-9+3=-16$, so we get the equation for the hyperplane

$$10x_1-5x_2+9x_3-3x_4=-16.$$

2. Find a parametrization of a plane in $\mathbb{R}^4$ going through $(0,1,0,1)$ and parallel to

$$H:\begin{cases} x_1+x_2-2x_3+x_4=-1 \\ 2x_1+3x_2-2x_3+x_4=5 \end{cases}$$

We get immediately a system of equations for the direction

$$\begin{cases} x_1+x_2-2x_3+x_4=0 \\ 2x_1+3x_2-2x_3+x_4=0 \end{cases}$$

And the free coefficients are $0+1+0+1=2, 0+3+0+1=4$, so we get the following system for the plane

$$\begin{cases} x_1+x_2-2x_3+x_4=2 \\ 2x_1+3x_2-2x_3+x_4=4 \end{cases}.$$

To find a parametrization we have to solve it

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 2 \\ 2 & 3 & -2 & 1 & 4 \end{bmatrix} \to \ldots \to \begin{bmatrix} 1 & 0 & -4 & 2 & 2 \\ 0 & 1 & 2 & -1 & 0 \end{bmatrix}$$

So the general solution (and a parametrization) is

$$\{(2+4x_3-2x_4,-2x_3+x_4,x_3,x_4)\colon x_3,x_4\in\mathbb{R}\}$$