Linear algebra, WNE, 2018/2019
meeting 19. – homework solutions

6 December 2018

Group 8:00

1. Let $W = \text{lin}((1,1,2))$. Find a basis of $W^\perp$ in $\mathbb{R}^3$.

   This is a space of solutions to $x_1 = -x_2 - 2x_3$, so we get basis $\{(-1,1,0),(-2,0,1)\}$.

2. Let $W = \text{lin}((1,0,1,0),(0,1,0,2),(2,-2,2,-4))$ and let V be the space of solutions to $x_1+2x_2-x_3+x_4 = 0$. W and V are subspaces of $\mathbb{R}^4$ with the standard scalar product. Find orthonormal bases of W and V.

   We start with W:

$$w_1 = (1,0,1,0),$$

$$w_2 = (0,1,0,2) - \frac{\langle (1,0,1,0),(0,1,0,2)\rangle}{\langle (1,0,1,0),(1,0,1,0)\rangle}(1,0,1,0) = (0,1,0,2) - \frac{0}{2}(1,0,1,0) = (0,1,0,2),$$

$$w_3 = (2,-2,2,-4) - \frac{\langle (1,0,1,0),(2,-2,2,-4)\rangle}{\langle (1,0,1,0),(1,0,1,0)\rangle}(1,0,1,0) - \frac{\langle (0,1,0,2),(2,-2,2,-4)\rangle}{\langle (0,1,0,2),(0,1,0,2)\rangle}(0,1,0,2) =$$

$$= (2,-2,2,-4) - \frac{4}{2}(1,0,1,0) - \frac{-10}{5}(0,1,0,2) = (0,0,0,0).$$

   So we get an orthogonal basis of W: $\{(1,0,1,0),(0,1,0,2)\}$, so we get orthonormal basis: $\left\{\frac{(1,0,1,0)}{\sqrt{2}}, \frac{(0,1,0,2)}{\sqrt{5}}\right\}$.
   Basis of V is $\{(-2,1,0,0),(1,0,1,0),(-1,0,0,1)\}$. We start the procedure

$$v_1 = (-2,1,0,0),$$

$$v_2 = (1,0,1,0) - \frac{\langle (-2,1,0,0),(1,0,1,0)\rangle}{\langle (-2,1,0,0),(-2,1,0,0)\rangle}(-2,1,0,0) = (1,0,1,0) + \frac{2}{5}(-2,1,0,0) = \frac{1}{5}(1,2,5,0),$$

$$v_3 = (-1,0,0,1) - \frac{\langle (-2,1,0,0),(-1,0,0,1)\rangle}{\langle (-2,1,0,0),(-2,1,0,0)\rangle}(-2,1,0,0) - \frac{\langle (1,2,5,0),(-1,0,0,1)\rangle}{\langle (1,2,5,0),(1,2,5,0)\rangle}(1,2,5,0) =$$

$$= (-1,0,0,1) - \frac{2}{5}(-2,1,0,0) + \frac{1}{30}(1,2,5,0) = \frac{1}{30}(-5,-10,5,30) = \frac{1}{6}(-1,-2,1,6).$$

   So we get the following orthogonal basis of V: $\{(-2,1,0,0),(1,2,5,0),(-1,-2,1,6)\}$, and orthonormal basis: $\left\{\frac{(-2,1,0,0)}{\sqrt{5}}, \frac{(1,2,5,0)}{\sqrt{30}}, \frac{(-1,-2,1,6)}{\sqrt{42}}\right\}$.

Group 9:45

1. Let $W = \text{lin}((1,1,3))$. Find a basis of $W^\perp$ in $\mathbb{R}^3$.

   It is a space of solutions to $x_1 = -x_2 - 3x_3$, so its basis is $\{(-1,1,0),(-3,0,1)\}$.

2. Let $W = \text{lin}((1,1,0,0),(0,0,1,2),(2,2,-2,-4))$, and let V be the basis of solutions to $x_1 - x_2 + 2x_3 + x_4 = 0$. V and W are subspaces of $\mathbb{R}^4$ with the standard scalar product. Find orthonormal bases of W and V.

   First W:

$$w_1 = (1,1,0,0),$$

$$w_2 = (0,0,1,2) - \frac{\langle (1,1,0,0),(0,0,1,2)\rangle}{\langle (1,1,0,0),(1,1,0,0)\rangle}(1,1,0,0) = (0,0,1,2) - \frac{0}{2}(1,1,0,0) = (0,0,1,2),$$

$$w_3 = (2,2,-2,-4) - \frac{\langle (1,1,0,0),(2,2,-2,-4)\rangle}{\langle (1,1,0,0),(1,1,0,0)\rangle}(1,1,0,0) - \frac{\langle (0,0,1,2),(2,2,-2,-4)\rangle}{\langle (0,0,1,2),(0,0,1,2)\rangle}(0,0,1,2) =$$

$$= (2,2,-2,-4) - \frac{4}{2}(1,1,0,0) - \frac{-10}{5}(0,0,1,2) = (0,0,0,0).$$

We get an orthogonal basis of W: $\{(1,1,0,0),(0,0,1,2)\}$, and so after normalization we get: $\left\{\frac{(1,1,0,0)}{\sqrt{2}}, \frac{(0,0,1,2)}{\sqrt{5}}\right\}$.
Basis V is $\{(1,1,0,0),(-2,0,1,0),(-1,0,0,1)\}$. The procedure goes as follows:

$$v_1 = (1,1,0,0),$$

$$v_2 = (-2,0,1,0) - \frac{\langle (1,1,0,0),(-2,0,1,0)\rangle}{\langle (1,1,0,0),(1,1,0,0)\rangle}(1,1,0,0) = (-2,0,1,0) + \frac{2}{2}(1,1,0,0) = (-1,1,1,0),$$

$$v_3 = (-1,0,0,1) - \frac{\langle (1,1,0,0),(-1,0,0,1)\rangle}{\langle (1,1,0,0),(1,1,0,0)\rangle}(1,1,0,0) - \frac{\langle (-1,1,1,0),(-1,0,0,1)\rangle}{\langle (-1,1,1,0),(-1,1,1,0)\rangle}(-1,1,1,0) =$$

$$= (-1,0,0,1) + \frac{1}{2}(1,1,0,0) - \frac{1}{3}(-1,1,1,0) = \frac{1}{6}(-1,1,-2,6).$$

So we get an orthogonal basis of V: $\{(1,1,0,0),(-1,1,1,0),(-1,1,-2,6)\}$, and after normalization: $\left\{\frac{(1,1,0,0)}{\sqrt{2}}, \frac{(-1,1,1,0)}{\sqrt{3}}, \frac{(-1,1,-2,6)}{\sqrt{42}}\right\}$.