Linear algebra, WNE, 2018/2019
meeting 17.

27 November 2018

Problems

1. For endomorphism $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2, \varphi((x,y)) = (3x+4y, 5x-2y)$ and bases $\mathcal{A}_1 = \{(4,1),(3,1)\}, \mathcal{A}_2 = \{(2,3),(5,8)\}, \mathcal{A}_3 = \{(4,2),(1,1)\}$ find matrices $\mathcal{A}_i = M(\varphi)_{\mathcal{A}_i}^{\mathcal{A}_i}$ and matrices C_{ij} satisfying $A_j = C_{ij}^{-1}A_iC_{ij}$ for $i,j = 1,2,3$.
2. For the following endomorphisms find eigenvalues and bases of eigenspaces related to each eigenvalue.
   - $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2, \varphi((x,y)) = (2x-y, -x+2y)$,
   - $\varphi\colon \mathbb{R}^4 \to \mathbb{R}^4, \varphi((x,y,z,t)) = (-6x-y+2z, 3x+2y+t, -14x-2y+5z, -t)$.
3. For the endomorphism $\varphi\colon V \to V$ check whether there exists a basis $\mathcal{A}$ of V which consists of eigenvectors of φ. If the answer is in the positive, give an example of such a basis and calculate $M(\varphi)_{\mathcal{A}}^{\mathcal{A}}$.
   - $V = \mathbb{R}^2, \varphi((a,b)) = (a-b, a+3b)$,
   - $V = \mathbb{R}^4, \varphi((a,b,c,d)) = (2a+4b, 5a+3b, c+d, 3c-d)$.
4. Check whether matrices $A_1 = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$ are diagonalizable. If so, find C_i, such that $C_i^{-1}A_iC_i$ is diagonal for $i = 1,2$.

Homework

Group 8:00

1. For the eigenvalues and bases of their eigenspaces of endomorphism $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3, \varphi((x,y,z)) = (x-y, x+3y+z, 2z)$.
2. Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3, \varphi((a,b,c)) = (3a, a+2b-c, a-b+2c)$. Check whether there exists a basis $\mathcal{A}$ of $\mathbb{R}^3$ which consists of eigenvectors of φ. If the answer is in the positive, give an example of such a basis and find $M(\varphi)_{\mathcal{A}}^{\mathcal{A}}$.
3. Check whether matrix $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ is diagonalizable. If it is, find a matrix C, such that $C^{-1}AC$ is a diagonal matrix.

Group 9:45

1. For the eigenvalues and bases of their eigenspaces of endomorphism $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3, \varphi((x,y,z)) = (x-z, 2y, x+y+3z)$.
2. Consider endomorphism $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3, \varphi((a,b,c)) = (2a-b+c, -a+2b+c, 3c)$. Check whether there exists a a basis $\mathcal{A}$ of $\mathbb{R}^3$ which consists of eigenvectors of φ. If the answer is in the positive, give an example of such a basis and find $M(\varphi)_{\mathcal{A}}^{\mathcal{A}}$.
3. Check whether $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ is diagonalizable. If it is, find a matrix C, such that $C^{-1}AC$ is a diagonal matrix.