Linear algebra, WNE, 2018/2019
meeting 13.

15 November 2018

Problems

1. Let $\mathcal{A} = \{(-2,1),(-1,1)\}, \mathcal{B} = \{(3,2),(2,-2)\}$, $\mathcal{C} = \{(1,0,1,0),(0,0,-1,0),(0,2,0,1),(0,1,0,1)\}$, and let $\phi, \varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ i $\psi\colon \mathbb{R}^2 \to \mathbb{R}^4$ be such that
   - $\psi((x,y)) = (x+y,-x,-3y,-x+2y)$,
   - $M(\phi)^{st}_{\mathcal{A}} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$,
   - $M(\varphi)^{\mathcal{B}}_{\mathcal{A}} = \begin{bmatrix} -1 & 0 \\ -2 & 3 \end{bmatrix}$,

   Find
   - $M(\mathrm{id})^{\mathcal{C}}_{st}$,
   - $M(\psi)^{st}_{st}$,
   - $M(\varphi)^{st}_{\mathcal{A}}$,
   - $M(\psi\circ(\varphi+3\phi))^{\mathcal{C}}_{\mathcal{A}}$,
   - the coordinates of $\psi(\varphi(v)+3\phi(v))$ in the basis $\mathcal{C}$, if vector v has coordinates $1,1$ with respect to $\mathcal{A}$.
2. Let $\mathcal{A} = \{(5,7,1),(4,0,0),(6,2,5)\}, \mathcal{B} = \{(1,-1,1),(0,1,6),(0,1,5)\}$. Find a matrix $C \in M_{3\times 3}(\mathbb{R})$ such that for every $\alpha \in \mathbb{R}^3$ we get the following. If a_1,a_2,a_3 are the coordinates of α with respect to $\mathcal{A}$, and b_1,b_2,b_3 are the coordinates of this vector with respect to $\mathcal{B}$, then
$$C\cdot\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$
3. Let $\mathcal{A} = \{(2,1),(1,1)\}, \mathcal{B} = \{(1,3),(0,1)\}, \mathcal{C} = \{(0,1),(1,4)\}$, and let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $M(\varphi)^{\mathcal{B}}_{\mathcal{A}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find $M(\varphi)^{\mathcal{C}}_{\mathcal{A}}$.

Homework

Group 8:00

1. Let $\mathcal{A} = \{(1,2,3),(2,1,0),(4,5,0)\}, \mathcal{B} = \{(2,1,2),(3,1,2),(2,1,3)\}$. Find a matrix $C \in M_{3\times 3}(\mathbb{R})$, such that for any vector $\alpha \in \mathbb{R}^3$ we get that if the coordinates of α with respect to $\mathcal{A}$ are a_1,a_2,a_3 , and b_1,b_2,b_3 are the coordinates of this vector with respect to $\mathcal{B}$, then
$$C\cdot\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$
2. Let $\mathcal{A} = \{(1,1,2),(1,2,1),(0,0,1)\}, \mathcal{B} = \{(2,1),(1,-1)\}, \mathcal{C} = \{(2,4),(-1,1)\}$ and let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $M(\varphi)^{\mathcal{B}}_{\mathcal{A}} = \begin{bmatrix} -1 & 3 & 2 \\ 5 & -3 & 4 \end{bmatrix}$. Find $M(\varphi)^{\mathcal{C}}_{\mathcal{A}}$.

Group 9:45

1. Let $\mathcal{A} = \{(2,1,0),(1,2,3),(4,5,0)\}, \mathcal{B} = \{(1,2,2),(1,2,3),(1,3,2)\}$. Find a matrix $C \in M_{3\times 3}(\mathbb{R})$, such that for any vector $\alpha \in \mathbb{R}^3$ we get the following. If a_1,a_2,a_3 are the coordinates of α with respect to $\mathcal{A}$, and b_1,b_2,b_3 are the coordinates of this vector with respect to $\mathcal{B}$, then
$$C\cdot\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$
2. Let $\mathcal{A} = \{(2,1,1),(1,2,1),(1,0,0)\}, \mathcal{B} = \{(1,-1),(2,1)\}, \mathcal{C} = \{(-1,1),(2,4)\}$ and let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $M(\varphi)^{\mathcal{B}}_{\mathcal{A}} = \begin{bmatrix} 1 & 3 & 2 \\ 5 & 3 & 4 \end{bmatrix}$. Find $M(\varphi)^{\mathcal{C}}_{\mathcal{A}}$.