Linear algebra, WNE, 2018/2019
meeting 12.

13 November 2018

Problems

1. Find the matrices of a linear transformation φ in the standard bases and also in bases $\mathcal{A}, \mathcal{B}$:
   - $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^2, \varphi((x_1, x_2, x_3)) = (x_1 - x_2 + 4x_3, -3x_1 + 8x_3), \mathcal{A} = \{(3,4,1),(2,3,1),(5,1,1)\}, \mathcal{B} = \{(3,1),(2,1)\}$,
   - $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^4, \varphi((x,y)) = (3x+y, x+5y, -x+4y, 2x+y), \mathcal{A} = \{(3,1),(4,2)\}$, $\mathcal{B} = \{(1,0,1,0),(0,1,1,1),(0,1,2,3),(0,0,0,1)\}$,
   - $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3, \varphi((x,y,z)) = (4x+y+z, 3x+2y+z, 3x+2y+z), \mathcal{A} = \{(3,1,1),(1,0,0),(5,1,0)\}, \mathcal{B} = \{(1,-1,1),(4,1,1),(2,0,1)\}$.
2. Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with matrix
$$M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$
in bases $\mathcal{A} = \{(3,1,1),(1,0,0),(5,1,0)\}$, $\mathcal{B} = \{(3,4,5),(4,1,1),(2,0,1)\}$. Find the formula for φ.
3. Let $\varphi\colon V \to W, \psi\colon W \to Z$ be linear transformations with $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 2 & 1 & 4 & 5 \\ 1 & 0 & 1 & 3 \end{bmatrix}$ and $M(\psi)_{\mathcal{B}}^{\mathcal{C}} = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 0 & 1 \end{bmatrix}$ in bases $\mathcal{A}, \mathcal{B}, \mathcal{C}$ of spaces V, W, Z, respectively. Let $\alpha \in V$ have coordinates $1, -1, 3, -2$ with respect to A. Find the coordinates of $\varphi(\alpha)$ with respect to $\mathcal{B}$, the coordinates of $(\psi \circ \varphi)(\alpha)$ with respect to $\mathcal{C}$ and the matrix $M(\psi \circ \varphi)_{\mathcal{A}}^{\mathcal{C}}$.

Homework

1. Find the matrix of linear transformation $\varphi\colon \mathbb{R}^4 \to \mathbb{R}^2, \varphi((a,b,c,d)) = (5a - 2b + 3c - d, 3a + 4b + 6d)$ in bases $\mathcal{A} = \{(2,1,0,1),(1,0,3,1),(2,1,1,3),(3,1,2,1)\}, \mathcal{B} = \{(5,2),(3,1)\}$.
2. Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with matrix
$$M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$$
in bases $\mathcal{A} = \{(-1,-1),(2,0)\}, \mathcal{B} = \{(1,1,1),(1,-1,-1),(4,3,2)\}$. Find the formula for φ.
3. Let $\mathcal{A} = \{(0,1,0),(1,2,3),(5,7,1)\}, \mathcal{B} = \{(0,1),(1,1)\}, \mathcal{C} = \{(2,1),(1,0)\}$ and let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation with matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$, and let $\psi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by formula $\psi((y_1, y_2)) = (y_1 - y_2, y_1 + y_2)$. Find:
   - $M(\psi \circ \varphi)_{\mathcal{A}}^{\mathcal{C}}$,
   - the formula for $\psi \circ \varphi$.