Linear algebra, WNE, 2018/2019
meeting 8.

25 October 2018

Problems

1. Let V and W be subspaces of $\mathbb{R}^5$ such that $V = \text{lin}((10, 3, 9+s, 1, 2-s), (4, 1, 6, 1, 1), (2, 1, -1, -1, -2))$, and W is the subspace of solutions of

$$\begin{cases} 3x_1 - 11x_2 + tx_3 - 8x_4 + x_5 = 0 \\ 2x_1 - 4x_2 - x_3 + 3x_4 - x_5 = 0 \\ x_1 - 5x_2 + x_3 - 6x_4 + x_5 = 0 \end{cases}$$

   Find $\dim V$ and $\dim W$ depending on $s, t \in \mathbb{R}$. Find s, t such that $V = W$.

2. Let V be the space described by the following system of equations:

$$\begin{cases} x + y + z + t + w = 0 \\ x - y + z - t + w = 0 \end{cases}$$

   Complete, if it is possible, the following systems of vectors to the basis of $\mathbb{R}^5$ using only vectors from V.

   - $(5, -1, 2, 1, 7), (2, 3, -6, -3, 4)$,
   - $(1, 2, 3, -2, -4), (6, 4, -5, -4, -1), (3, -2, -14, 2, 11)$.

3. Find systems of equations describing the following linear subspaces:

   - $\text{lin}((4, 1, 2, -3), (2, 3, 1, -9), (2, -1, 1, 3), (6, 4, 3, -12))$,
   - $\text{lin}((5, 1, 9, 0, 2), (5, 2, -2, 5, -1), (4, 1, 5, 1, 1))$.

4. Find an example of a vector $\alpha \in \{(x, y, z) \in \mathbb{R}^3 \colon x + 2y - z = 0\}$, such that $(2, 1, 3), (1, 4, 5), \alpha$ is a basis of $\mathbb{R}^3$.

5. Does there exist $\beta \in \text{lin}((3, 5, 8), (-1, 3, 2))$ such that $(2, 1, 3), (1, 4, 5), \beta$ is a basis of $\mathbb{R}^3$? If so find an example of such vector β.