Linear Algebra, WNE, 2018/2019
meeting 6. – homework solutions

18 October 2018

Group 8:00

1. Find a basis and dimension of space

$$\text{lin}((2,1,3),(3,5,-1),(3,-2,13),(7,7,7),(-4,-9,5))$$

and space

$$\text{lin}((2,7,-1,2,0),(3,1,4,2,0),(4,-5,9,2,0),(5,15,2,6,0)).$$

We transform the matrix to an echelon form:

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 7 & 7 & 7 \\ -4 & -9 & 5 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_4} \begin{bmatrix} 7 & 7 & 7 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 2 & 1 & 3 \\ -4 & -9 & 5 \end{bmatrix} \xrightarrow{w_1 \cdot \frac{1}{7}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 2 & 1 & 3 \\ -4 & -9 & 5 \end{bmatrix} \xrightarrow{w_2 - 3w_1, w_3 - 3w_1, w_4 - 2w_1, w_5 + 4w_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -4 \\ 0 & -5 & 10 \\ 0 & -1 & 1 \\ 0 & -5 & 9 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{1}{2}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & -5 & 10 \\ 0 & -1 & 1 \\ 0 & -5 & 9 \end{bmatrix} \xrightarrow{w_3 + 5w_2, w_4 + w_2, w_5 + 5w_2}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{w_3 \leftrightarrow w_5} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{w_4 - w_5} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So we get basis $\{(1,1,1),(0,1,-2),(0,0,-1)\}$, and dimension equals 3.

In the second space we write the columns in order x_4, x_1, x_2, x_3, x_5:

$$\begin{bmatrix} 2 & 2 & 7 & -1 & 0 \\ 2 & 3 & 1 & 4 & 0 \\ 2 & 4 & -5 & 9 & 0 \\ 6 & 5 & 15 & 2 & 0 \end{bmatrix} \xrightarrow{w_2 - w_1, w_3 - w_1, w_4 - 3w_1} \begin{bmatrix} 2 & 2 & 7 & -1 & 0 \\ 0 & 1 & -6 & 5 & 0 \\ 0 & 2 & -12 & 10 & 0 \\ 0 & -1 & -6 & 5 & 0 \end{bmatrix} \xrightarrow{w_3 - 2w_2, w_4 + w_2}$$

$$\begin{bmatrix} 2 & 2 & 7 & -1 & 0 \\ 0 & 1 & -6 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & 10 & 0 \end{bmatrix} \xrightarrow{w_3 \leftrightarrow w_4} \begin{bmatrix} 2 & 2 & 7 & -1 & 0 \\ 0 & 1 & -6 & 5 & 0 \\ 0 & 0 & -12 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So we get basis $\{(2,7,-1,2,0),(1,-6,5,0,0),(0,-12,10,0,0)\}$, and dimension equals 3.

2. Find a basis and dimension of a space described by the following system of equations.

$$\begin{cases} 3x + y + z - 4t = 0 \\ 7x + 3y + 5z + 2t = 0 \\ 2x + y + 2z + 3t = 0 \end{cases}$$

We solve this system of equations (order of columns y, x, z, t, w):

$$\begin{bmatrix} 1 & 3 & 1 & -4 \\ 3 & 7 & 5 & 2 \\ 1 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{w_2 - 3w_1, w_3 - w_1}$$

$$\begin{bmatrix} 1 & 3 & 1 & -4 \\ 0 & -2 & 2 & 14 \\ 0 & -1 & 1 & 7 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{-1}{2}} \begin{bmatrix} 1 & 3 & 1 & -4 \\ 0 & 1 & -1 & -7 \\ 0 & -1 & 1 & 7 \end{bmatrix} \xrightarrow{w_3 + w_2}$$

$$\begin{bmatrix} 1 & 3 & 1 & -4 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{w_1 - 3w_2} \begin{bmatrix} 1 & 0 & 4 & 17 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We get the general solution (remembering the order of columns) $(z + 7t, -4z - 17t, z, t)$, so we get a basis $\{(1, -4, 1, 0), (7, -17, 0, 1)\}$, and the dimension equals 2.

Group 9:45

1. Find a basis and dimension of space

$$\text{lin}((2, 1, 4), (3, 5, -1), (3, -2, 13), (7, 7, 7), (-4, -9, 6))$$

and space

$$\text{lin}((2, 7, -1, 2, 6), (3, 1, 4, 2, 2), (4, -5, 9, 2, -2), (5, 15, 2, 6, 14)).$$

We transform the following matrix to an echelon form.

$$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 7 & 7 & 7 \\ -4 & -9 & 6 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_4} \begin{bmatrix} 7 & 7 & 7 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 2 & 1 & 4 \\ -4 & -9 & 6 \end{bmatrix} \xrightarrow{w_1 \cdot \frac{1}{7}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & -1 \\ 3 & -2 & 13 \\ 2 & 1 & 4 \\ -4 & -9 & 6 \end{bmatrix} \xrightarrow{w_2 - 3w_1, w_3 - 3w_1, w_4 - 2w_1, w_5 + 4w_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -4 \\ 0 & -5 & 10 \\ 0 & -1 & 2 \\ 0 & -5 & 10 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{1}{2}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & -5 & 10 \\ 0 & -1 & 2 \\ 0 & -5 & 10 \end{bmatrix} \xrightarrow{w_3 + 5w_2, w_4 + w_2, w_5 + 5w_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So we get a basis $\{(1, 1, 1), (0, 1, -2)\}$, and dimension equals 2.

In the second case write out the columns on order x_4, x_1, x_2, x_3, x_5:

$$\begin{bmatrix} 2 & 2 & 7 & -1 & 6 \\ 2 & 3 & 1 & 4 & 2 \\ 2 & 4 & -5 & 9 & -2 \\ 6 & 5 & 15 & 2 & 14 \end{bmatrix} \xrightarrow{w_2 - w_1, w_3 - w_1, w_4 - 3w_1} \begin{bmatrix} 2 & 2 & 7 & -1 & 6 \\ 0 & 1 & -6 & 5 & -4 \\ 0 & 2 & -12 & 10 & -8 \\ 0 & -1 & -6 & 5 & -4 \end{bmatrix} \xrightarrow{w_3 - 2w_2, w_4 + w_2}$$

$$\begin{bmatrix} 2 & 2 & 7 & -1 & 6 \\ 0 & 1 & -6 & 5 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & 10 & -8 \end{bmatrix} \xrightarrow{w_3 \leftrightarrow w_4} \begin{bmatrix} 2 & 2 & 7 & -1 & 6 \\ 0 & 1 & -6 & 5 & -4 \\ 0 & 0 & -12 & 10 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So the basis is $\{(2,7,-1,2,6),(1,-6,5,0,-4),(0,-12,10,0,-8)\}$, and dimension equals 3.

2. Fins a basis and dimension of a space described by the following system of equations.

$$\begin{cases} 7x + 3y + 5z + 2t + 8w = 0 \\ 3x + y + z - 4t + 6w = 0 \\ 2x + y + 2z + 3t + w = 0 \end{cases}$$

We solve this system of equations (order of columns: y, x, z, t, w):

$$\begin{bmatrix} 3 & 7 & 5 & 2 & 8 \\ 1 & 3 & 1 & -4 & 6 \\ 1 & 2 & 2 & 3 & 1 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_2} \begin{bmatrix} 1 & 3 & 1 & -4 & 6 \\ 3 & 7 & 5 & 2 & 8 \\ 1 & 2 & 2 & 3 & 1 \end{bmatrix} \xrightarrow{w_2 - 3w_1, w_3 - w_1}$$

$$\begin{bmatrix} 1 & 3 & 1 & -4 & 6 \\ 0 & -2 & 2 & 14 & -10 \\ 0 & -1 & 1 & 7 & -5 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{-1}{2}} \begin{bmatrix} 1 & 3 & 1 & -4 & 6 \\ 0 & 1 & -1 & -7 & 5 \\ 0 & -1 & 1 & 7 & -5 \end{bmatrix} \xrightarrow{w_3 + w_2}$$

$$\begin{bmatrix} 1 & 3 & 1 & -4 & 6 \\ 0 & 1 & -1 & -7 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{w_1 - 3w_2} \begin{bmatrix} 1 & 0 & 4 & 17 & -9 \\ 0 & 1 & -1 & -7 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So the general solution takes the following form (recall the order of columns) $(z + 7t - 5w, -4z - 17t + 9w, z, t, w)$, so we get a basis $\{(1,-4,1,0,0),(7,-17,0,1,0),(-5,9,0,0,1)\}$, and dimension equals 3.