Linear algebra, WNE, 2018/20198
meeting 4. – homework solutions

1 October 2018

Group 8:00

1. Check whether the following subsets of $\mathbb{R}^2$ satisfy any of the conditions from the definition of a linear subspace.

   - $\{(x, y) : x = 0 \text{ or } y = x\}$,
     Does not satisfy the addition condition, because $(1, 0)$ and $(1, 1)$ are in this set, but $(0, 1) + (1, 1) = (1, 2)$ is not. It does satisfy the multiplication condition, because both $c\cdot(x, 0) = (cx, 0)$ and $c\cdot(x, x) = (cx, cx)$ are elements of the subset.
   - $\{(x, y) : x^2 + 4y^2 = 4xy\}$.
     The above equation is equivalent to $x - 2y = 0$, so $x = 2y$. This set satisfies both the conditions because both $(x, 2x) + (y, 2y) = (x + y, 2x + 2y)$ and $c \cdot (x, 2x) = (cx, 2cx)$ are the elements of this subset.

2. Are the following systems of vectors:

   - $(3, 2, 0), (-1, 0, 2), (4, 2, 1)$,
   - $(4, 2, 1, -2), (5, 0, -1, 6), (1, 1, 2, 2)$

   independent?

   - We transform appropriate matrix into an echelon form:

     $$\begin{bmatrix} 3 & 2 & 0 \\ -1 & 0 & 2 \\ 4 & 2 & 1 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_2} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} \xrightarrow{w_2 + 3w_1, w_4 + 4w_1} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 6 \\ 0 & 2 & 9 \end{bmatrix} \xrightarrow{w_3 - w_2} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

     Last vector is a non-zero vector, thus the system is linearly independent.
   - Similarly,

     $$\begin{bmatrix} 4 & 2 & 1 & -2 \\ 5 & 0 & -1 & 6 \\ 1 & 1 & 2 & 2 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_3} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 5 & 0 & -1 & 6 \\ 4 & 2 & 1 & -2 \end{bmatrix} \xrightarrow{w_2 - 5w_1, w_3 - 4w_1}$$

     $$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -5 & -11 & -4 \\ 0 & -2 & -7 & -10 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{-1}{5}} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2,2 & 0,8 \\ 0 & -2 & -7 & -10 \end{bmatrix} \xrightarrow{w_3 + 2w_2} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2,2 & 0,8 \\ 0 & 0 & -4,8 & -8,4 \end{bmatrix}$$

     Last vector is a non-zero vector, thus the system is linearly independent.

Group 9:45

1. Check whether the following subsets of $\mathbb{R}^2$ satisfy any of the conditions from the definition of a linear subspace.

   - $\{(x, y) : x = 0 \text{ or } y = 0\}$,
     The set does not satisfy the addition condition, because $(1, 0)$ and $(0, 1)$ are in this set, but $(0, 1) + (1, 0) = (1, 1)$ is not. It satisfies the multiplication condition, because both $c \cdot (x, 0) = (cx, 0)$ and $c \cdot (0, y) = (0, cy)$ are in this set.

- $\{(x, y)\colon x^2 + y^2 = 2xy\}$.
  This equation is equivalent to $x - y = 0$, so $x = y$. This set is a linear subspace, because both $(x, x) + (y, y) = (x + y, x + y)$, and $c \cdot (x, x) = (cx, cx)$ are in the set for any x.

2. Are the following systems of vectors:

   - $(3, 2, 1), (-1, 0, 2), (4, 2, 2)$,
   - $(4, 2, 1, -2, 3), (5, 0, -1, 6, 1), (1, 1, 2, 2, 0)$

   independent?

   - We transform appropriate matrix into an echelon form:

     $$\begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 2 \\ 4 & 2 & 2 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_2} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 2 & 2 \end{bmatrix} \xrightarrow{w_2 + 3w_1, w_4 + 4w_1} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 7 \\ 0 & 2 & 8 \end{bmatrix} \xrightarrow{w_3 - w_2} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

     Last vector is a non-zero vector, thus the system is linearly independent.

   - Similarly,

     $$\begin{bmatrix} 4 & 2 & 1 & -2 & 3 \\ 5 & 0 & -1 & 6 & 1 \\ 1 & 1 & 2 & 2 & 0 \end{bmatrix} \xrightarrow{w_1 \leftrightarrow w_3} \begin{bmatrix} 1 & 1 & 2 & 2 & 0 \\ 5 & 0 & -1 & 6 & 1 \\ 4 & 2 & 1 & -2 & 3 \end{bmatrix} \xrightarrow{w_2 - 5w_1, w_3 - 4w_1}$$

     $$\begin{bmatrix} 1 & 1 & 2 & 2 & 0 \\ 0 & -5 & -11 & -4 & 1 \\ 0 & -2 & -7 & -10 & 3 \end{bmatrix} \xrightarrow{w_2 \cdot \frac{-1}{5}} \begin{bmatrix} 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 2.2 & 0.8 & 0.2 \\ 0 & -2 & -7 & -10 & 3 \end{bmatrix} \xrightarrow{w_3 + 2w_2}$$

     $$\begin{bmatrix} 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 2.2 & 0.8 & 0.2 \\ 0 & 0 & -4.8 & -8.4 & 3.4 \end{bmatrix}$$

     Last vector is a non-zero vector, thus the system is linearly independent.