Linear algebra, WNE, 2018/2019
meeting 3. – homework solutions

9 October 2018

Group 8:00

1. For which real numbers $s \in \mathbb{R}$ the following system of equations is inconsistent?

$$\begin{cases} 2x_1 + 2x_2 - 4x_3 = 6 \\ -x_1 + 3x_2 + 2x_3 = 1 \\ -2sx_1 + sx_2 + 4x_3 = -3 \end{cases}$$

We transform the matrix of this system into echelon form.

$$\left[\begin{array}{ccc|c} 2 & 2 & -4 & 6 \\ -1 & 3 & 2 & 1 \\ -2s & s & 4 & -3 \end{array}\right] \xrightarrow{w_1 \leftrightarrow w_2} \left[\begin{array}{ccc|c} -1 & 3 & 2 & 1 \\ 2 & 2 & -4 & 6 \\ -2 & s & 4 & -3 \end{array}\right] \xrightarrow{w_2 + 2w_1, w_3 - 2sw_1}$$

$$\left[\begin{array}{ccc|c} -1 & 3 & 2 & 1 \\ 0 & 8 & 0 & 8 \\ 0 & -5s & -4s+4 & -2s-3 \end{array}\right] \xrightarrow{w_2 \cdot \frac{1}{8}}$$

$$\left[\begin{array}{ccc|c} -1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -5s & -4s+4 & -2s-3 \end{array}\right] \xrightarrow{w_3 + 5sw_2} \left[\begin{array}{ccc|c} -1 & 3 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -4s+4 & 3s-3 \end{array}\right]$$

A system is inconsistent if we have zeroes in the last row except from the last place. But $-4s + 4 = 0$, if $s = 1$, but then also $3s - 3 = 0$, so there is no real number s for which this system is inconsistent.

Group 9:45

1. For which real numbers $s \in \mathbb{R}$ the following system of equations is inconsistent?

$$\begin{cases} -4x_1 + 2x_2 + 2x_3 = 6 \\ -2x_1 - 3x_2 + x_3 = -1 \\ -4x_1 - 2sx_2 + 4sx_3 = -4 \end{cases}$$

We transform the matrix of this system into echelon form.

$$\left[\begin{array}{ccc|c} -4 & 2 & 2 & 6 \\ -2 & -3 & 1 & -1 \\ -4 & -2s & 4s & -4 \end{array}\right] \xrightarrow{w_1 \leftrightarrow w_2} \left[\begin{array}{ccc|c} -2 & -3 & 1 & -1 \\ -4 & 2 & 2 & 6 \\ -4 & -2s & 4s & -4 \end{array}\right] \xrightarrow{w_2 - 2w_1, w_3 - 2w_1}$$

$$\left[\begin{array}{ccc|c} -2 & -3 & 1 & -1 \\ 0 & 8 & 0 & 8 \\ 0 & -2s+6 & 4s-2 & -2 \end{array}\right] \xrightarrow{w_2 \cdot \frac{1}{8}}$$

$$\left[\begin{array}{ccc|c} -2 & -3 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & -2s+6 & 4s-2 & -2 \end{array}\right] \xrightarrow{w_3 + (2s-6)w_2} \left[\begin{array}{ccc|c} -2 & -3 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4s-2 & -8+2s \end{array}\right]$$

A system is inconsistent if we have zeroes in the last row except from the last place. But $4s - 2 = 0$, so $s = \frac{1}{2}$ and then we have -7 as the free coefficient, and the system is inconsistent in this case.