

# Linear algebra, WNE, 2018/2019 meeting 2.

4 October 2018

## Problems

1. Which of the following systems of equations are homogeneous? Which are inconsistent? Which have exactly one solution?

$$U_1 : \begin{cases} 2x_1 - x_2 = 1 \\ x_1 + 2x_2 = 8 \end{cases},$$

$$U_2 : \begin{cases} x_1 + 2x_2 + 4x_3 + x_4 = 0 \\ -3x_1 + x_2 + 3x_3 + 5x_4 = 0 \\ 5x_1 + 2x_2 + 7x_3 = 0 \end{cases},$$

$$U_3 : \begin{cases} x_1 - x_2 + x_3 = 2 \\ 2x_2 - x_3 = 8 \\ -x_1 + x_2 - x_3 = 0 \\ -x_1 + 8x_2 + 7x_3 = -4 \end{cases},$$

$$U_4 : x_1 + 2x_2 - x_3 + x_4 = 5.$$

2. Which tuple  $(-1, 1, 1, -1), (2, 3, 1, 4), (4, -3, 2, 1), (4, 0, -3, \frac{1}{2})$  satisfy the following system of equations?

$$\begin{cases} 3x_1 + 2x_2 + 4x_3 + 2x_4 = 1 \\ 7x_1 + 5x_2 + 9x_3 + 4x_4 = 3 \\ 5x_1 - 3x_2 + 7x_3 + 4x_4 = 1 \end{cases}$$

3. Find the general solution to the following system of equations.

$$\begin{cases} x_1 + 3x_2 + x_3 + 5x_4 = 2 \\ 2x_1 + 7x_2 + 9x_3 + 2x_4 = 4 \\ 4x_1 + 13x_2 + 11x_3 + 12x_4 = 8 \end{cases}$$

4. Find the general solution to the following system of equations.

$$\begin{cases} 2x_1 - x_2 + x_3 + 2x_4 + 3x_5 = 2 \\ 6x_1 - 3x_2 + 2x_3 + 4x_4 + 5x_5 = 3 \\ 6x_1 - 3x_2 + 4x_3 + 8x_4 + 13x_5 = 9 \\ 4x_1 - 2x_2 + x_3 + x_4 + 2x_5 = 1 \end{cases}$$

5. For which real numbers  $t \in \mathbb{R}$  tuple  $(1, t, 3, 2t)$  is a solution to the following system of equations?

$$\begin{cases} 3x_1 + 2x_2 + x_3 - x_4 = 6 \\ 2x_1 + 5x_2 - 3x_3 - 2x_4 = 5 \\ x_1 - 4x_2 + 5x_3 + 2x_4 = 16 \end{cases}$$

6. For which real numbers  $s \in \mathbb{R}$  the system of equations

$$\begin{cases} x_1 + 2x_2 + 2x_3 + 3x_4 = 2 \\ 3x_1 + 5x_2 + 4x_3 + 8x_4 = 7 \\ x_1 + 3x_2 + 4x_3 + 4x_4 = s \end{cases}$$

is consistent?

7. Let  $w(x)$  be a polynomial of degree 3 satisfying conditions  $w(0) = -1, w(1) = 3, w(2) = 7, w(-1) = -5$ . Find the coefficients of  $w(x)$ .
8. Three brothers Antoni, Bonifacy and Cezary added up their ages, and found out that they are in total 100 years old. Moreover, 10 years ago, the age of Antoni was equal to the sum of the age of Bonifacy and half of the age of Cezary. Is it possible, that the sum of the age of Antoni from 25 years ago multiplied by 4, and the age of Cezary now equals 100 years as well?

## Homework

1. Find the general solution to the following system of equations.

$$\begin{cases} 6x + 4y + 5z + 2w + 3t = 1 \\ 3x + 2y + 4z + w + 2t = 3 \\ 3x + 2y - 2z + w = -7 \\ 9x + 6y + z + 3w + 2t = 2 \end{cases}.$$

2. Does there exist a polynomial  $w(x)$  of degree 2 such that  $w(-2) = 2, w(3) = 10$  which has both real roots such that their product equals  $-3$ ? *Hint: use Vieta's formulas.*
3. For which real numbers  $t \in \mathbb{R}$  tuple  $(t^2, -1, 1, -t^2, 1)$  is a solution of the following system of equations.

$$\begin{cases} 7x_1 - 5x_2 - 3x_3 + 5x_4 - 5x_5 = -1 \\ 9x_1 + 8x_2 - 9x_3 + 2x_4 + 11x_5 = 1 \\ -4x_1 + 6x_2 + 2x_3 - x_4 + 9x_5 = 2 \end{cases}$$