On strong shape and homology theory of continuous maps

Anzor Beridze

Shota Rustavely Batumi State University, Department of Mathematics, Batumi, Georgia

anzorberidze@yahoo.com

The strong shape theory of continuous maps of general topological spaces, so called strong fiber shape theory is constructed. The current work is motivated by papers [1-4], [6]. In particular, using of Theorem 3.7 of [2] and methods developed in this paper the spectral and strong homology groups of continuous maps are defined and studied [4], [5]. It is shown that spectral and strong homology groups of continuous maps are fiber shape and strong fiber shape invariant, respectively. The relationship of the constructed groups is found. In particular for each continuous maps of compact Housdorf spaces the Milnor short exact sequence is constructed (cf. [7] and [8]): for each continuous map $f : X \to Y$ of compact metric spaces there exists the short exact sequence:

$$0 \to \lim^1 H_{n+1}(\mathrm{f}_\alpha; G) \to \overline{\mathbf{H}}_n(f; \mathrm{G}) \to \check{H}_n(f; G) \to 0,$$

were $f = \lim f_\alpha$. Besides, we have gave the example of the map $f : X \to Y$ for which $\lim^1 H_{n+1}(\mathrm{f}_\alpha; G) \neq 0$ and therefore $\overline{\mathbf{H}}_n(f; \mathrm{G}) \neq \check{H}_n(f; G)$.

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